

Testing the interaction model with cosmological data and gamma-ray bursts

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We use the new gamma-ray bursts (GRBs) data, combined with the baryon acoustic oscillation(BAO) observation from the spectroscopic Sloan Digital Sky Survey (SDSS) data release, the newly obtained A parameter at $z = 0.6$ from the WiggleZ Dark Energy Survey, the cosmic microwave background (CMB) observations from the 7-Year Wilkinson Microwave Anisotropy Probe (WMAP7) results, and the type Ia supernovae (SNeIa) from Union2 set, to constrain a phenomenological model describing possible interactions between dark energy and dark matter, which was proposed to alleviate the coincidence problem of the standard Λ CDM model. By using the Markov Chain Monte Carlo (MCMC) method, we obtain the marginalized 1σ constraints $\Omega_m = 0.2886 \pm 0.0135$, $r_m = -0.0047 \pm 0.0046$, and $w_X = -1.0658 \pm 0.0564$. We also consider other combinations of these data for comparison. These results show that: (1) the energy of dark matter is slightly transferring to that of dark energy; (2) even though the GRBs+BAO+CMB data present less stringent constraints than SNe+BAO+CMB data do, the GRBs can help eliminate the degeneracies among parameters.

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1. INTRODUCTION

A lot of astrophysical and cosmological observations have indicated that the universe is undergoing an accelerating expansion and great efforts have been made to understand the driving force behind the cosmic acceleration [1–15]. This gives birth to the construction of a strange dark energy with negative pressure, which may contribute to interpret the present accelerated expansion. The most simple candidate for these uniformly distributed dark energy is considered to be in the form of vacuum energy density or cosmological constant (Λ). However, despite its simplicity, the simple cosmological constant is always entangled with the coincidence problem: Why the present matter density ρ_m , which decreases with the expansion of our universe with a^{-3} , is comparable with the dark energy density ρ_Λ , which does not change with the cosmic expansion of our universe? In order to relieve the coincidence problem, other dynamic dark energy models were proposed in the past decades, including quintessence [16, 17], phantom [18, 19], k-essence [20, 21], as well as quintom model [22–24]. However, the nature of dark energy is still unknown. The other presumption is naturally considered that energy is exchanged between dark energy and dark matter through interaction.

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We assume that dark energy and dark matter exchange energy through interaction term Q , namely

$$\dot{\rho}_X + 3H\rho_X(1 + w_X) = -Q, \quad (1)$$

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (2)$$

and the total energy conservation equation expresses as $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0$, where $\rho_{tot} = \rho_X + \rho_m$. Because the format of interaction term still can not be determined from fundamental physics, many literature have extensively considered various forms of the interaction term Q [25–37]

In this work, we consider the simplest case for convenience, i.e. [38]

$$Q = 3r_m H \rho_m, \quad (3)$$

where r_m is a dimensionless constant: $r_m = 0$ indicates that there is no interaction between dark energy and dark matter; the energy is transferred from dark matter to dark energy when $r_m < 0$, and from dark energy to dark matter when $r_m > 0$. We assume that the equation of state (EoS) of dark energy, $w_X \equiv p_X/\rho_X$, is a constant in a spatially flat FRW metric. In this case, Eq. (1) reads

$$\dot{\rho}_X + 3H\rho_X(1 + w_X) = -3r_m H \rho_m. \quad (4)$$

Combining Eq. (2) and Eq. (4), we can get

$$\rho_m = \rho_{m0}(1 + z)^{3(1 - r_m)} = \rho_{m0}a^{-3(1 - r_m)} \quad (5)$$

By using Eq. (5), we obtain

$$\rho_X = Aa^{-3(1 + w_X)} - \frac{r_m \rho_{m0}}{(r_m + w_X)} a^{-3(1 - r_m)}, \quad (6)$$

where A is an integral constant. Inserting it into the Friedmann equation

$$H^2 = \kappa^2(\rho_m + \rho_X)/3, \quad (7)$$

where $\kappa^2 = 8\pi G$ with G the gravitational constant, and this integral constant can be determined with $H(z = 0) = H_0$.

The corresponding Hubble parameter is

$$E^2(z) = (H/H_0)^2 = \frac{w_X \Omega_m}{(r_m + w_X)} (1 + z)^{3(1 - r_m)} + \left(\frac{1 - w_X \Omega_m}{(r_m + w_X)}\right) (1 + z)^{3(1 + w_X)}. \quad (8)$$

Recently the simple phenomenological interacting scenario has been constrained from several cosmological observations [34, 39, 40]. Also, the Gamma-ray bursts (GRBs) have been proposed as distance indicators and regarded as a complementary cosmological probe of the universe at high redshifts [41–55]. In contrast to supernovae, the high energy photons in the gamma-ray band are nearly unaffected by dust extinction. Therefore, those observed high-redshift Gamma-Ray Bursts (GRBs) at $0.1 < z < 8.1$ may constitute a complementary probe to fill the "desert" between the redshifts of SNIa and CMB. Nevertheless, due to the lack of a low-redshift GRBs at $z < 0.1$, there is a circularity problem in the direct use of GRBs [47]. Some statistical methods have been proposed to alleviate this problem, such as the scatter method [43], the luminosity distance method [43] and the Bayesian method [44]. Other methods trying to avoid the circularity problem have been proposed in Ref. [56, 57]. Using the cosmology-independent calibration

method proposed in Ref. [58], [59] have obtained 59 calibrated high-redshift GRBs called "Hymnium" GRBs sample out of 109 long GRBs with the well-known Amati relation. Therefore, it may be rewarding to test the coupling between dark sectors with this newly obtained 59 GRBs deprived of the circularity problem.

With this aim, in this paper, we adopt the Markov Chain Monte Carlo(MCMC) technique to constrain one interaction model from the latest observational data. To reduce the uncertainty and put tighter constraint on the value of the coupling, we combine the GRBs data with the joint observations such as the 557 Union2 SNeIa dataset [5], the CMB observation from the Wilkinson Microwave Anisotropy Probe (WMAP7) [9] results, and the model independent new A parameter from the baryon acoustic oscillation (BAO) measurements [60], the two BAO distance ratios at $z = 0.2$ and $z = 0.35$ from the spectroscopic Sloan Digital Sky Survey (SDSS) data release7 (DR7) galaxy sample [61]. This paper is organized as follows: we introduce the observational data in Section 2. The numerical analysis results are discussed in Section 3, and the main conclusions are summarized in Section 4.

2. OBSERVATIONAL DATA

The 59 calibrated Hymnium GRBs and the 557 Union2 SNeIa data sets are given in term of the distance modulus $\mu(z)$. Theoretically, the distance modulus can be calculated as

$$\mu = 5 \log \frac{d_L}{Mpc} + 25 = 5 \log_{10} H_0 d_L - \mu_0, \quad (9)$$

where $\mu_0 = 5 \log_{10} [H_0 / (100 km/s/Mpc)] + 42 \cdot 38$, and the luminosity distance d_L can be calculated using $d_L = \frac{(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}$. The χ^2 value of the observed distance moduli can be calculated by

$$\chi_\mu^2 = \sum_{i=1}^N \frac{[\mu(z_i) - \mu_{obs}(z_i)]^2}{\sigma_{\mu i}^2}, \quad (10)$$

where $\mu_{obs}(z_i)$ is the observed distance modulus for the SNe Ia or GRBs at redshift z_i with error $\sigma_{\mu i}$; $\mu(z_i)$ is the theoretical value of the distance modulus calculated from Eq. (9). The nuisance parameter h is marginalized with a flat prior, after which we get [62]

$$\chi_\mu^2 = \sum_{i=1}^N \frac{\alpha_i^2}{\sigma_{\mu i}^2 t^2} - \frac{(\sum_{i=1}^N \alpha_i / \sigma_{\mu i}^2 - \ln 10 / 5)^2}{\sum_{i=1}^N 1 / \sigma_{\mu i}^2} - 2 \ln \left(\frac{\ln 10}{5} \sqrt{\frac{2\pi}{\sum_{i=1}^N 1 / \sigma_i^2}} \right),$$

where $\alpha_i = \mu_{obs}(z_i) - 25 - 5 \log_{10} H_0 d_L$. In this part the radiation component of the total density is neglected, because its contribution in low redshifts is negligible.

For the BAO observation, we use the new BAO A parameter at $z = 0.6$, with the measured value $A = 0.452 \pm 0.018$ [60] from the WiggleZ Dark Energy Survey. It can be calculated as the following:

$$A = \sqrt{\Omega_m} \frac{H_0 D_V(z=0.6)}{z=0.6} = \frac{\sqrt{\Omega_m}}{0.6} \left[\frac{0.6}{E(0.6)} \frac{1}{|\Omega_k|} \text{sinn}^2 \left(\sqrt{|\Omega_k|} \int_0^{0.6} \frac{dz}{E(z)} \right) \right]^{1/3}, \quad (11)$$

Now we can add our χ^2 obtained before with

$$\chi_{BAOa}^2(\mathbf{p}) = \left(\frac{A - 0.452}{0.018} \right)^2. \quad (12)$$

Notice that the BAO A parameter does not depend on the baryon density $\Omega_b h^2$ or the Hubble constant h . The radiation density which does depend on h is negligible to the Hubble parameter $E(z)$.

In addition to the above A parameter, we also consider the BAO distance ration (d_z) at $z = 0.2$ and $z = 0.35$ from SDSS data release 7 (DR7) galaxy sample [61]. The BAO distance ration can be expressed as

$$d_z = \frac{r_s(z_d)}{D_V(z)}, \quad (13)$$

where the effective distance is given by [11]

$$D_V(z) = \left[\frac{d_L^2(z)}{(1+z)^2} \frac{z}{H(z)} \right]^{1/3}; \quad (14)$$

and the drag redshift z_d is fitted as [63]

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}} [1 + b_1(\Omega_b h^2)^{b_2}], \quad (15)$$

$$\begin{aligned} b_1 &= 0.313(\Omega_m h^2)^{-0.419} [1 + 0.607(\Omega_m h^2)^{0.674}], \\ b_2 &= 0.238(\Omega_m h^2)^{0.223}, \end{aligned} \quad (16)$$

The comoving sound horizon is

$$r_s(z) = \int_z^\infty \frac{c_s(z) dz}{E(z)}, \quad (17)$$

where the sound speed $c_s(z) = 1/\sqrt{3[1 + \bar{R}_b/(1+z)]}$, and $\bar{R}_b = 3\Omega_b h^2/(4 \times 2.469 \times 10^{-5})$. The χ^2 value of BAO observation can be expressed as [61]

$$\chi_{\text{BAO}}^2 = \Delta \mathbf{P}_{\text{BAO}}^T \mathbf{C}_{\text{BAO}}^{-1} \Delta \mathbf{P}_{\text{BAO}}, \quad (18)$$

where $\Delta \mathbf{P}_{\text{BAO}} = \mathbf{P}_{\text{th}} - \mathbf{P}_{\text{obs}}$, \mathbf{P}_{obs} is the observed distance ration. $\mathbf{C}_{\text{BAO}}^{-1}$ is the corresponding inverse covariance matrix.

For the CMB observation, we use the derived dataset from the WMAP7 measurement, including the acoustic scale (l_a), the shift parameter R , and the redshift of recombination z_* [9]. The acoustic scale can be expressed as

$$l_a = \pi \frac{\Omega_k^{-1/2} \text{sinn}[\Omega_k^{1/2} \int_0^{z_*} \frac{dz}{E(z)}]/H_0}{r_s(z_*)}, \quad (19)$$

where $r_s(z_*) = H_0^{-1} \int_{z_*}^\infty c_s(z)/E(z) dz$ is the comoving sound horizon at photon-decoupling epoch. The shift parameters can be expressed as

$$R(z_*) = \frac{\sqrt{\Omega_m}}{\sqrt{|\Omega_k|}} \text{sinn} \left(\sqrt{|\Omega_k|} \int_0^{z_*} \frac{dz}{E(z)} \right). \quad (20)$$

The decoupling redshift z_* is fitted by [64],

$$z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{g_2}], \quad (21)$$

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}. \quad (22)$$

The χ^2 for CMB can be expressed then as

$$\chi_{\text{CMB}}^2 = \Delta \mathbf{P}_{\text{CMB}}^T \mathbf{C}_{\text{CMB}}^{-1} \Delta \mathbf{P}_{\text{CMB}}, \quad (23)$$

where $\Delta \mathbf{P}_{\text{CMB}} = \mathbf{P}_{\text{th}} - \mathbf{P}_{\text{obs}}$, \mathbf{P}_{obs} is the observed dataset, and the $\mathbf{C}_{\text{CMB}}^{-1}$ is corresponding inverse covariance matrix.

Data	w_X	r_m	Ω_m	$H_0/100$	χ^2/dof
all	$-1.0658 \pm 0.0564(1\sigma)$	$-0.0047 \pm 0.0046(1\sigma)$	$0.2886 \pm 0.0135(1\sigma)$	$0.7369 \pm 0.0391(1\sigma)$	0.9140
all-GRBs	$-1.0657 \pm 0.0562(1\sigma)$	$-0.0047 \pm 0.0046(1\sigma)$	$0.2886 \pm 0.0135(1\sigma)$	$0.7367 \pm 0.0389(1\sigma)$	0.9686
all-SNe	$-1.2757 \pm 0.2358(1\sigma)$	$-0.0062 \pm 0.0045(1\sigma)$	$0.2649 \pm 0.0288(1\sigma)$	$0.7984 \pm 0.0803(1\sigma)$	0.3580

TABLE I: The marginalized 1σ errors of the parameters w_X , r_m , Ω_m , and H_0 for the phenomenological scenario, as well as χ^2/dof , obtained from the combinations of the data sets GRBs+SNe+BAO+CMB (all), SNe+BAO+CMB (all-GRBs), and GRBs+BAO+CMB (all-SNe).

3. CONSTRAINT ON THE PHENOMENOLOGICAL INTERACTING SCENARIO

The model parameters are determined by applying the minimum likelihood method of χ^2 fit. Basically, the model parameters are determined by minimizing

$$\chi^2 = \chi_{GRBs}^2 + \chi_{SNe}^2 + \chi_{CMB}^2 + \chi_{BAO}^2 + \chi_{BAOa}^2. \quad (24)$$

We apply the Monte Carlo Markov Chain (MCMC) method [65] with 8 chains and obtain the marginalized 1σ constraints: $\Omega_m = 0.2886 \pm 0.0135$, $r_m = -0.0047 \pm 0.0046$, and $w_X = -1.0658 \pm 0.0564$. We show the marginalized probability distribution of each parameter and the marginalized 2D confidence contours of parameters in Figure 1. For comparison, fitting results from the combinations of SNe+BAO+CMB and GRBs+BAO+CMB are shown in Figure 2. Because CMB and BAO data tightly constrain the matter density Ω_m , and therefore help improve the constraints on dark energy property [15], so these data are taken as priors in our treatment and are combined with other data to test the constraining power of GRBs and SNe data. It is shown that the plots in Figure 1 and the left panel of Figure 2 are almost the same, suggesting that the current GRBs data are consistent with other observations, although they contribute little to the existing constraints. Comparing the constraints from SNe data with those from GRBs data in Figure 2, we see that the degeneracies between Ω_m and w_X , Ω_m and r_m are different although the current GRBs data give larger errors on w_X . Therefore, GRBs data have the potential to help constrain the model parameters w_X and r_m by combining with SNe data. The best-fit values of the parameters along with their 1σ uncertainties from all three different combinations mentioned above, are explicitly presented in Table I. In general, for the three different joint data sets, at the 1σ confidence region, the energy is seen transferred from dark matter to dark energy, and the concordance ΛCDM ($w_X = -1$, $r_m = 0$) model can not be excluded.

From Figure 1, we see that the coupling parameter r_m is correlated with all other parameters Ω_m , w_X and H_0 . However, $r_m = 0$ is within the 1σ confidence region. To see the point clearly, we fix $r_m=0$ and obtain the marginalized 1σ uncertainties of the other model parameters from the combination of all observational data (SNe+CMB+BAO+GRBs): $\Omega_m = 0.2863 \pm 0.0130$, $w_X = -1.0436 \pm 0.0509$, and $H_0/100 = 0.6994 \pm 0.0118$. The constraints on the model without interaction ($r_m = 0$) are consistent with those with interaction at 1σ level, and w_X shifts toward -1 when $r_m = 0$ so that it is consistent with ΛCDM model at 1σ level.

4. CONCLUSIONS

In this paper we constrain an interacting dark energy model using the new GRBs, the Union2 SNe, CMB, and BAO data sets. By adopting the MCMC approach we obtained the marginalized 1σ errors of each parameter:

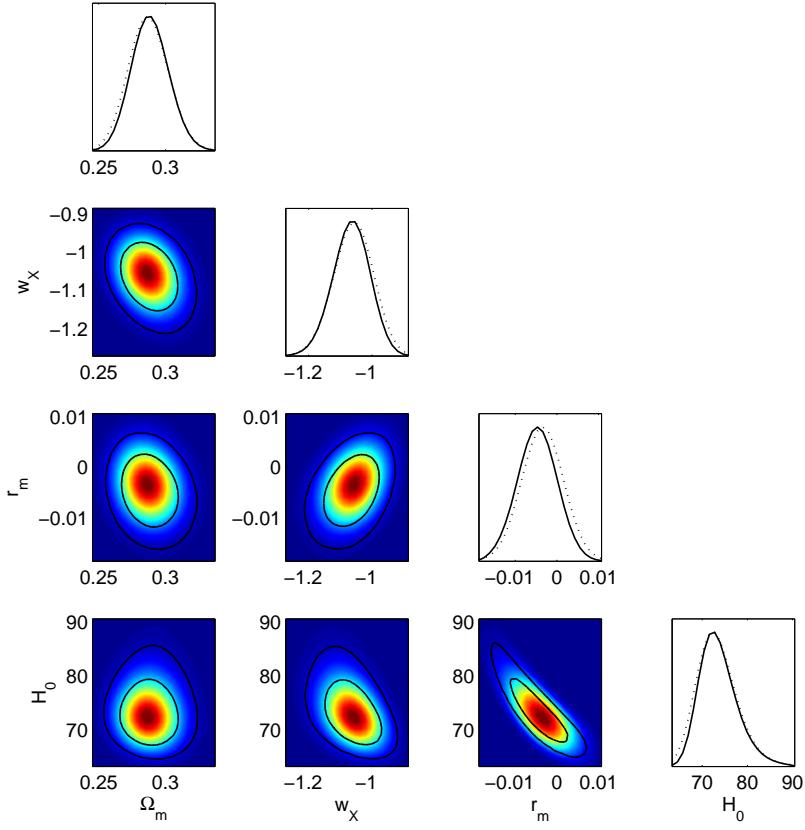


FIG. 1: The marginalized probability distributions, and the marginalized 1σ and 2σ confidence contours of the parameters w_X , r_m , Ω_m , and H_0 in the phenomenological interacting scenario, from the combinations of all observational data SNe+BAO+CMB+GRBs.

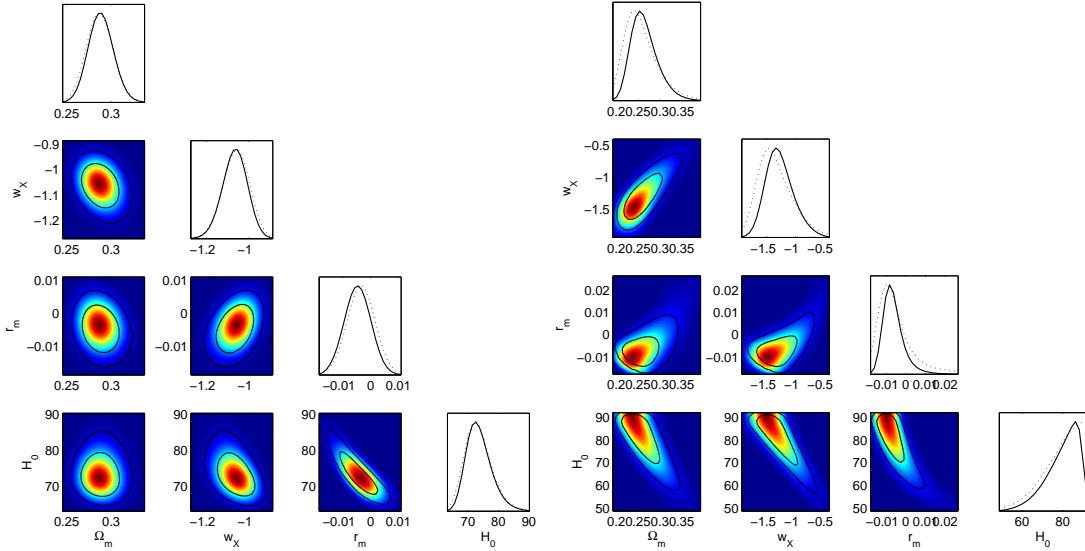


FIG. 2: The same as in Figure 1, but the constraints in the left panel are from the data combinations SNe+BAO+CMB (all-GRBs) and the constraints in the right panel are from the data combinations GRBs+BAO+CMB (all-SNe).

$\Omega_m = 0.2886 \pm 0.0135$, $r_m = -0.0047 \pm 0.0046$, and $w_X = -1.0658 \pm 0.0564$.

In order to test the constraining power of the observational GRBs data, we compared the results from the combinations of GRBs+BAO+CMB with those from SNe+BAO+CMB. We find that the constraints from the current GRBs data are less stringent than those from SNe data. The error bars of w_X , Ω_m and H_0 from GRBs+BAO+CMB are roughly twice of those from SNe+BAO+CMB, although the error bar of r_m is roughly the same as shown in Table I and Figure 2. However, the directions of degeneracies between Ω_m and w_X , Ω_m and r_m are different, so GRBs data have the potential to help tighten the constraint on the parameters Ω_m , w_X and r_m if the measurement precision of the data is improved in the future. By fitting the model to all the observational data combined, we find that the energy slightly transfers from dark matter to dark energy at 1σ region. We also note that the coupling parameter r_m is correlated with all other model parameters Ω_m , w_X and H_0 , and $r_m = 0$ is within the 1σ confidence region. The constraints on the model without interaction ($r_m = 0$) are consistent with those with interaction at 1σ level, and the value of w_X shifts up a little when $r_m = 0$. In conclusion, the concordance Λ CDM model still remains a good fit to the observational data.

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